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Chiral primary cubic interactions from pp-wave supergravity

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Abstract: We explicitly construct cubic interaction light-cone hamiltonian for the chiral primary system involving the metric fields and the self-dual four-form fields in the IIB pp-wave supergravity. The background fields representing pp-waves exhibit $\mathrm{SO}(4)_{\perp}\times\mathrm{SO}(4)_{\parallel}\times Z_2$ invariance. It turns out that the interaction hamiltonian is precisely the same as that for the dilaton-axion system, except for the fact that the chiral primary system fields have the opposite parity to that of the dilaton-axion fields under the Z_2 transformation that exchanges two SO(4)'s.

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While the IIB string theory in flat space-time exhibits SO(8) invariance, pp-wave backgrounds of refs. [1, 2] possess $\mathrm{SO}(4)_\perp \times \mathrm{SO}(4)_{||} \times Z_2$ invariance. The consequence of this fact at the level of supergravity spectrum is that there are two extra scalars s and $\bar{s}$ under $\mathrm{SO}(4)_\perp \times \mathrm{SO}(4)_{||}$ on top of the dilaton and axion that are scalars under SO(8) (τ and $\bar{\tau}$ with the energy $m^2 = 4f$). They come from the trace part of the $\mathrm{SO}(4)_\perp$ subspace metric, that is related to the $\mathrm{SO}(4)_{||}$ subspace metric via the SO(8) traceless condition, and the four-form RR gauge field along the $\mathrm{SO}(4)_\perp$, which is related to the $\mathrm{SO}(4)_{||}$ part through the self-duality condition. At the free theory level, it is known that they combine to produce chiral primary with $m^2 = 0$ and anti-chiral primary with $m^2 = 8f$ [3]. Recalling that the Z_2 part of the symmetry is an element of the SO(8) that exchanges $\mathrm{SO}(4)_\perp$ and $\mathrm{SO}(4)_{||}$, the dilaton and axion, being SO(8) scalars, have even parity under the Z_2. When viewed from this angle, chiral primaries should have odd parity. The confirmation of this expectation at the supergravity interaction level is the purpose of this paper. Specifically, we construct the cubic interaction part of the light-cone hamiltonian for the chiral primary system starting from the covariant IIB supergravity. It turns out that thus obtained interaction hamiltonian is precisely the same as that for the dilaton-axion system [4, 5], except for the fact that s and $\bar{s}$ (τ and $\bar{\tau}$) have odd (even) parity under the exchange of the two $\mathrm{SO}(4)'$s.[1] This result is interesting at least from two perspectives. First, in view of the apparent difference in the SO(8) tensor structure of the two systems, this agreement is not entirely trivial. Generically, the treatment of the self-dual four-form fields in curved backgrounds is a non-trivial problem [7], but the pp-wave backgrounds provide us with a tractable setting. Secondly, the ground state[2] of the string theory in pp-wave backgrounds should be given odd parity; supergravity can be obtained from string theory via a smooth deformation of parameters, and the discrete charge assignment should not change under this deformation.

The bosonic sector of type-IIB supergravity involves a dilaton, an axion, a graviton, two antisymmetric 2-form fields, and a self-dual antisymmetric four-form field. We are interested in the system of the gravity field $g_{\mu\nu}$ and the self-dual antisymmetric 4-form gauge field $a_{\mu\nu\rho\sigma}$. The classical dynamics of these fields are described by Einstein equations

$$R_{\mu\nu} = \frac{1}{4!} F_{\mu\rho\sigma\tau\lambda} F_\nu{}^{\rho\sigma\tau\lambda} \,, \tag{1}$$

and the four-form gauge field equations

$$\nabla_\mu F^{\mu\nu\rho\sigma\tau} = 0 \,, \tag{2}$$

where the five-form field strength tensor is defined as

$$F_{\mu\nu\rho\sigma\tau} \equiv \partial_{[\mu} a_{\nu\rho\sigma\tau]} \,, \qquad \mu, \nu, \ldots = +, -, 1, 2, \ldots, 8 \,. \tag{3}$$

In addition, the self-duality condition [3]

$$F_{\mu\nu\rho\sigma\tau} = -\frac{1}{5!} \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma\tau\mu'\nu'\rho'\sigma'\tau'} F^{\mu'\nu'\rho'\sigma'\tau'} \,, \tag{4}$$

[1] See [6] for related discussions.

[2] In the string field theory side [8, 9], this state is the supersymmetric ground state $|vac\rangle$ satisfying $Q|vac\rangle = \bar{Q}|vac\rangle = 0$ with zero energy. In the dual gauge theory side [10, 11], this corresponds to the operator $\mathrm{Tr} Z^J$ of ref. [2]. The 'string states' in the weak coupling limit of the dual gauge theory has the energy (slightly) larger than $2f$.

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where

$$\epsilon_{+-12345678} = -1\,, \qquad \epsilon_{0123456789} = -1\,, \tag{5}$$

should be imposed. In fact, it is well known that the self-duality condition (4) implies the equations of motion (2).

To obtain the pp-wave backgrounds, we turn on constant background condensate f for certain components of the five-form field strength

$$F_{+1234} = F_{+5678} = 2f\,, \tag{6}$$

and set others to zero. The nontrivial component of the Einstein equations (1) is

$$R_{++} = 8f^2\,, \tag{7}$$

solving which determines the metric to be

$$ds^2 = 2dx^+dx^- - f^2x_I^2dx^+dx^+ + dx^Idx^I\,, \tag{8}$$

where $I, J, \ldots = 1, 2, \ldots, 8$ denote SO(8) indices. The pp-wave metric (8) and the constant condensate for the field strength (6) constitute our background configuration in what follows.

Our analysis of the gravity sector will closely follow that of Goroff and Schwarz [12]. Adopting their notations, the metric fluctuations are parametrized as follows:

$$(g)_{\mu\nu} = \begin{pmatrix} g_{++} & g_{+-} & g_{+J} \\ g_{-+} & g_{--} & g_{-J} \\ g_{I+} & g_{I-} & g_{IJ} \end{pmatrix}, \tag{9}$$

where

$$g_{++} = -f^2x_I^2 + h_{++}\,, \qquad g_{+-} = e^{\varphi}\,, \qquad g_{IJ} = e^{\psi}\gamma_{IJ} \qquad \text{with} \quad \det(\gamma_{IJ}) = 1\,. \tag{10}$$

To implement the light-cone gauge fixing, we choose the nine components of the metric to zero

$$g_{--} = g_{-I} = 0\,. \tag{11}$$

Using the remaining gauge invariance out of the ten original diffeomorphisms, we impose a relation between g_{+-} and $\det(g_{IJ})$

$$\varphi = \frac{1}{2}\psi\,. \tag{12}$$

This condition simplifies the $(--)$-component of Ricci tensor

$$R_{--} = 4\left[\partial_-^2\psi + \frac{1}{2}(\partial_-\psi)^2 - \partial_-\varphi\partial_-\psi\right] - \frac{1}{4}\partial_-\gamma_{IJ}\partial_-\gamma^{IJ}\,, \tag{13}$$

where γ^{IJ} satisfies $\gamma^{IJ}\gamma_{JK} = \delta^I{}_K$. Since we are interested in the chiral primary system, we turn on only one gravitational fluctuation out of the 35 physical graviton degrees of freedom γ_{IJ}

$$\gamma_{IJ} = \begin{pmatrix} e^hI_4 & 0 \\ 0 & e^{-h}I_4 \end{pmatrix}, \tag{14}$$

where I_4 is 4×4 unit matrix. This is the general form of the physical field fluctuations satisfying $\det(\gamma_{IJ}) = 1$ that are scalars under $\mathrm{SO}(4)_\perp \times \mathrm{SO}(4)_{||}$. Note that the field h has negative parity under the Z_2 transformation that exchanges the two SO(4)'s, namely, under the interchange of the upper and lower blocks. The expression for the inverse metric

$$(g)^{\mu\nu} = \begin{pmatrix} 0 & e^{-\psi/2} & 0 \\ e^{-\psi/2} & (f^2 x_I^2 - h_{++})e^{-\psi} + g_{+I}\gamma^{IJ}g_{J+} & -e^{-\frac{3}{2}\psi}g_{+I}\gamma^{IJ} \\ 0 & -e^{-\frac{3}{2}\psi}g_{+J}\gamma^{JI} & e^{-\psi}\gamma^{IJ} \end{pmatrix} \tag{15}$$

will be useful in what follows.

For the self-dual four-form gauge field $a_{\mu\nu\rho\sigma}$, we choose the light-cone gauge condition

$$a_{-\mu\nu\rho} = 0\,, \tag{16}$$

which is possible due to the presence of the gauge transformations for the four-form gauge field. After the imposition of the self-duality condition (4), there are 35 physical degrees of freedom. Among these, we again turn on fluctuations that are scalars under $\mathrm{SO}(4)_\perp \times \mathrm{SO}(4)_{||}$ transformations. Consequently, all mixed components such as $a_{ijkl'}$ ($a_{ij'k'l'}$) or $a_{ijk'l'}$ vanish, where $i, j, \ldots = 1, 2, 3, 4$ are vector indices of $\mathrm{SO}(4)_{||}$ and $i', j', \ldots = 5, 6, 7, 8$ are vector indices of $\mathrm{SO}(4)_\perp$. The nonvanishing components a_{1234} and a_{5678} are constrained by the (-1234)-component (or, equivalently, (-5678)-component) of the self-dual equation (4):

$$\partial_- a_{1234} = -e^{4h}\partial_- a_{5678}\,. \tag{17}$$

In the derivation of (17), we have used $\sqrt{-g} = e^{9\psi/2}$ and the explicit form of the upper index metric (15). We note that when the relative sign between the $(+1234)$-component and the $(+5678)$-component in (6) is positive, the relative sign appearing in (17) is necessarily negative. Consequently, when we turn off the metric fluctuation h, the field $\partial_- a_{1234} = -\partial_- a_{5678} = \partial_- a$ has negative parity under the Z_2 transformation.

Our ultimate goal is to identify the quadratic action for the chiral primary system (h, a) and to construct cubic interactions among them. We thus solve the gauge constraints from both the Einstein equations (1) and the self-dual equations (4) to express the unphysical fields in terms of the physical fields. To be specific, $(--)$-, $(+-)$-, $(-I)$-components of the Einstein equations are solved to express ten auxiliary fields (h_{++}, g_{+I}, ψ) of the covariant metric (9), or equivalently (g^{--}, g^{-I}, ψ) of the contravariant metric (15) in terms of (h, a). With the help of eqs. (12)–(14), the Einstein equation (12) for the variation of g^{--} leads to

$$\begin{aligned} R_{--} &= 4\partial_-^2\psi + 2(\partial_- h)^2 \\ &= \left[(\partial_- a_{1234})^2 + (\partial_- a_{5678})^2 + \mathcal{O}(3) + \cdots\right], \end{aligned} \tag{18}$$

up to quadratic terms. We thus obtain

$$\psi = \frac{1}{4}\left(\frac{1}{\partial_-^2}\right)\left[-2(\partial_- h)^2 + (\partial_- a_{1234})^2 + (\partial_- a_{5678})^2\right] + \mathcal{O}(3) + \cdots\,. \tag{19}$$

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The operator $1/\partial_-$ is to be understood as the inverse momentum in the momentum basis, as is conventional in the light-cone quantization [12]. We also mention that one should carefully use integration by parts; for the cubic terms, for example, it is easier to understand it as using the momentum conservation conditions, such as $p_1^+ = -(p_2^+ + p_3^+)$. There can in principle be subtle zero mode effects, which often play an important role in light-cone approaches (see, for example, [13]). Our final answer, however, is Lorentz invariant when we set $f = 0$. Noting the fact that the terms involving f also have the ∂_- derivative and thereby undetectable by the the zero mode parts, the demonstration of the Lorentz invariance of the answer when $f = 0$ should be sufficient.

Up to quadratic terms, $(-I)$- and $(-+)$-components of the Einstein equations (1) are

$$\begin{aligned} R_{-I} &= \frac{1}{2}\partial_-^2\left(g^{-I} - \Delta_{IJ}\frac{\partial_J}{\partial_-}h\right) + 2\partial_- h\partial_I h + \frac{15}{4}\partial_I\partial_-\psi + \mathcal{O}(3) + \cdots \\ &= \frac{1}{4!}\partial_- a_{JKLM}\partial_I a_{JKLM} - \frac{1}{3!}\partial_- a_{+JKL}\partial_- a_{IJKL} + \mathcal{O}(3) + \cdots , \end{aligned} \tag{20}$$

where

$$\Delta_{IJ} = \begin{pmatrix} I_4 & 0 \\ 0 & -I_4 \end{pmatrix}_{IJ} . \tag{21}$$

For brevity, we simply list the following expression for the metric g^{-I} up to the linear terms

$$g^{j-} = \frac{\partial_j}{\partial_-}h + \mathcal{O}(2) + \cdots , \qquad g^{j'-} = -\frac{\partial_{j'}}{\partial_-}h + \mathcal{O}(2) + \cdots . \tag{22}$$

In the case of $(+-)$-component, we solve

$$\begin{aligned} R_{+-} &= \frac{1}{2}\partial_-^2\left(g^{--} - f^2x_I^2 + \frac{\partial_I}{\partial_-}g^{-I}\right) - \\ &\quad - \frac{1}{2}\Bigg[\frac{1}{2}\partial_-^2\left(g^{-I}g^{-I} - 2f^2x_I^2\psi\right) - \partial_I\left(\partial_- hg^{-J}\Delta_{JI} + \frac{1}{2}\partial_I\psi\right) - \\ &\qquad - 9\partial_-\partial_+\psi - 4\partial_+ h\partial_- h + \mathcal{O}(3) + \cdots\Bigg] \\ &= 2f(\partial_- a_{1234} + \partial_- a_{5678}) + \\ &\quad + \Bigg[-8fh(\partial_- a_{1234} - \partial_- a_{5678}) + (\partial_+ a_{1234}\partial_- a_{1234} + \partial_+ a_{5678}\partial_- a_{5678}) - \\ &\qquad - \frac{1}{4!}(\partial_- a_{1234}\epsilon_{ijkl}\partial_i a_{+jkl} + \partial_- a_{5678}\epsilon_{i'j'k'l'}\partial_{i'} a_{+j'k'l'}) - \\ &\qquad - \frac{1}{3!}\left((\partial_- a_{+ijk})^2 + (\partial_- a_{+i'j'k'})^2\right)\Bigg] + \mathcal{O}(3) + \cdots , \end{aligned} \tag{23}$$

to determine g^{--}. Inserting the metric component (22) into (23), we obtain an expression of g^{--} up to linear order

$$g^{--} = f^2x_I^2 - \frac{1}{\partial_-^2}\left[(\partial_i^2 - \partial_{i'}^2)h - 4f\partial_-(a_{1234} + a_{5678})\right] + \mathcal{O}(2) + \cdots . \tag{24}$$

The expression (24) for g^{--} apparently involves the four-form gauge field at the linear order, but using (17) transmutes the term to the quadratic order, making it irrelevant when we are allowed to neglect the quadratic and higher order terms in g^{--}. In fact, when investigating the self-dual gauge field equations, the expressions listed in (19), (22), and (24) are enough to determine the lagrangian up to cubic terms.

For the gauge sector, the self-duality condition (4) reproduces the wave equations (2) and, consequently, we will concentrate on the self-duality condition. We first solve F_{-ijkl} component equation (17) up to the quadratic terms to obtain

$$
\begin{aligned}
a_{1234} &= a + 2\frac{1}{\partial_-}(h\partial_- a) + \mathcal{O}(3) + \cdots\,, && (25)\\
a_{5678} &= -a + 2\frac{1}{\partial_-}(h\partial_- a) + \mathcal{O}(3) + \cdots\,. && (26)
\end{aligned}
$$

The equation (17) actually requires the sum of the coefficients of the second terms in (25) and (26) to be four. It will turn out later that this symmetric distribution yields us the lagrangian that is easiest to quantize; only in this case, the interaction dependent quadratic contribution to the canonical momentum is absent. It is worthwhile to note that the solutions (25) and (26) are analogous to the nonlocal field redefinition

$$
\chi \to \chi + 2\frac{1}{\partial_-}(\phi\partial_-\chi) \tag{27}
$$

introduced in [4] to investigate the dilaton ϕ and axion χ system.

The self-duality for the F_{+-ijk} components corresponds to the Gauss' law constraint part of (2). Utilizing it, we can express the unphysical $(+ijk)$- and $(+i'j'k')$-components of the four-form field in terms of h and a:

$$
\begin{aligned}
a_{+ijk} &= \epsilon_{ijkl}\frac{1}{\partial_-}\left[\partial_l a + \left(3h\partial_l a + \left(\frac{\partial_l}{\partial_-}h\right)\partial_- a - 2\frac{\partial_l}{\partial_-}(h\partial_- a)\right)\right] + \mathcal{O}(3) + \cdots\,, && (28)\\
a_{+i'j'k'} &= \epsilon_{i'j'k'l'}\frac{1}{\partial_-}\left[-\partial_{l'} a + \left(3h\partial_{l'} a + \left(\frac{\partial_{l'}}{\partial_-}h\right)\partial_- a - 2\frac{\partial_{l'}}{\partial_-}(h\partial_- a)\right)\right] + \mathcal{O}(3) + \cdots\,, && (29)
\end{aligned}
$$

where $\epsilon_{1234} = 1$ and $\epsilon_{5678} = 1$. This completes the determination of the relevant unphysical fields in terms of the physical fields.

Our next task is the construction of the lagrangian that correctly reproduces the equations of motion for a and h fields. The $(+1234)$-component, or equivalently $(+5678)$-component, does the job. With the help of self-dual solutions in eqs. (25), (26) and auxiliary fields in eqs. (28), (29), the difference of the two self-duality equations

$$
\partial_-\left(F_{+1234} - F_{+5678}\right) = \partial_-\left[\sqrt{-g}(F^{-5678} - F^{-1234})\right]\,, \tag{30}
$$

leads to an equation for a:

$$
\begin{aligned}
(\Box a - 8f\partial_- h) + \Bigg\{\Bigg[&-2\frac{\partial_i^2}{\partial_-}(h\partial_- a) - 2\partial_-\left(h\frac{\partial_i^2}{\partial_-}a\right) + 3\partial_i(h\partial_i a) + \partial_i\left(\partial_- a\frac{\partial_i}{\partial_-}h\right) + \\
&+ \partial_-\left(\partial_i a\frac{\partial_i}{\partial_-}h\right) - \partial_-\left(\partial_- a\frac{\partial_i^2}{\partial_-^2}h\right)\Bigg] - [i \to i']\Bigg\} + \mathcal{O}(3) + \cdots = 0\,, \qquad (31)
\end{aligned}
$$

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where $\Box$ denotes d'Alembertian in the pp-wave background

$$\Box = 2\partial_+\partial_- + \partial_I^2 + f^2 x_I^2 \partial_-^2 \,. \tag{32}$$

We have verified that (31) is consistent with the second order equation (2) up to quadratic terms. Note that the linear terms are a linearized wave equation for a and the quadratic terms represent cubic interaction of the type haa at the lagrangian level. Furthermore, (31) is invariant under the transformation $i \to i'$, $i' \to i$, $a \to -a$, $h \to -h$ under which both the linear terms and the quadratic terms change sign. The sum of both equations,

$$F_{+1234} + F_{+5678} = \sqrt{-g}(F^{-5678} + F^{-1234}) \,, \tag{33}$$

reproduces the linear part of eq. (31) so it provides a consistency check:

$$\left[h + \partial_- h \left(\frac{1}{\partial_-}\right)\right] (\Box a - 8f\partial_- h) + \mathcal{O}(3) + \cdots = 0 \,. \tag{34}$$

From (31), we can read off the lagrangian. The quadratic terms are

$$\mathcal{L}_{aa} = \frac{1}{\kappa^2} a\Box a \,, \tag{35}$$

$$\mathcal{L}_{ah} = \frac{16}{\kappa^2} fh\partial_- a \,, \tag{36}$$

where the choice of overall numerical normalization will be explained later. For the cubic interactions, we obtain

$$\begin{aligned} \mathcal{L}_{aah} = &-\frac{1}{\kappa^2}\left[4\frac{1}{\partial_-}(h\partial_- a)\partial_i^2 a + 3h\partial_i a\partial_i a + 2\partial_- a\partial_i a\frac{\partial_i}{\partial_-}h - \partial_- a\partial_- a\frac{\partial_i^2}{\partial_-^2}h\right] + \\ &+ \left\{i \to i',\ h \to -h,\ a \to -a\right\} . \end{aligned} \tag{37}$$

What has been left undetermined so far are the terms involving only h's, $\mathcal{L}_{hh}$ and $\mathcal{L}_{hhh}$, which require a separate analysis of the Einstein-Hilbert action or its equations of motion.

For the gravity sector, since we have already obtained all the quadratic and cubic terms involving a, it is sufficient to consider pure gravity case and set $a = 0$. In this case, we can directly adopt the procedures of ref. [12]. The only difference is that the background metric is that of the pp-wave backgrounds, not the flat metric. We obtain the following result:

$$\mathcal{L}_{hh} = \frac{1}{\kappa^2} h\Box h \,, \tag{38}$$

and that of the cubic interactions

$$\begin{aligned} \mathcal{L}_{hhh} = &-\frac{1}{\kappa^2}\left[4\frac{1}{\partial_-}(h\partial_- h)\partial_i^2 h + 3h\partial_i h\partial_i h + 2\partial_- h\partial_i h\frac{\partial_i}{\partial_-}h - \partial_- h\partial_- h\frac{\partial_i^2}{\partial_-^2}h\right] + \\ &+ \left\{i \to i',\ h \to -h,\ a \to -a\right\} . \end{aligned} \tag{39}$$

In all, the total lagrangian is given by a sum of five terms

$$\mathcal{L} = \mathcal{L}_{hh} + \mathcal{L}_{aa} + \mathcal{L}_{ah} + \mathcal{L}_{hhh} + \mathcal{L}_{haa} \,. \tag{40}$$

Several comments should be in order. First, the numerical normalization of the $\mathcal{L}_{hh}$, which determines the numerical normalization of the $\mathcal{L}_{hhh}$ term, has been chosen to reproduce the known quadratic level equations of motion in ref. [3]. Secondly, the mass scale f appears only in the d'Alembertian part of the quadratic terms. In fact, the action $S_{hh}+S_{hhh}$ is of the same form as that of Goroff and Schwarz [12], expanded up to cubic terms using (14), except for the f-dependent term in the d'Alembertian. The disappearance of the f-dependence in the cubic terms is consistent with the case of dilaton-axion system [4]. Thirdly, the resulting light-cone hamiltonian from (40) agrees with the (zero mode) string field theory analysis of [8] as well.

Introducing the chiral primary field $s = h + ia$ and its complex conjugate $\bar{s} = h - ia$, we derive the light-cone hamiltonian via a Legendre transform :

$$H(\bar{s}, s) = H_2 + H_3 \,. \tag{41}$$

Specific form of the quadratic hamiltonian is

$$H_2 = \frac{1}{\kappa^2}\int dx^- d^8x_I \left[f^2 x_I^2 \partial_-\bar{s}\partial_- s + \partial_I\bar{s}\partial_I s + 4if(\bar{s}\partial_- s - s\partial_-\bar{s})\right]. \tag{42}$$

The cubic interactions do not involve light-cone time derivatives (a consequence of the choice made in (25) and (26)) and the cubic hamiltonian is obtained by a sign flip of the cubic lagrangian in eqs. (37) and (39) :

$$\begin{aligned} H_3 = \frac{2}{\kappa^2}\int dx^- d^8x_I \Bigg\{ & \left[s\partial_i s\partial_i\bar{s} - \frac{1}{2}\frac{\partial_i}{\partial_-}(s\partial_- s)\partial_i\bar{s} - \frac{1}{2}\frac{\partial_i}{\partial_-}(s\partial_-\bar{s})\partial_i s\right] - \\ & - \frac{1}{4}\left[s\partial_i s\partial_i\bar{s} - \partial_i s\partial_-\bar{s}\frac{\partial_i}{\partial_-}s - \partial_- s\partial_i\bar{s}\frac{\partial_i}{\partial_-}s + \partial_- s\partial_-\bar{s}\frac{\partial_i^2}{\partial_-^2}s\right] + \\ & + \left(i \to i'\,,\ s \to -s\,,\ \bar{s} \to -\bar{s}\right)\Bigg\} + \{\text{c.c.}\}\,, \end{aligned} \tag{43}$$

where *c.c.* stands for the complex conjugation. The terms of the type sss and $\bar{s}\bar{s}\bar{s}$ get canceled when adding the contributions from $\mathcal{L}_{hhh}$ and $\mathcal{L}_{haa}$. Note that, in order to work out dynamical issues and structure of constraints systematically, we should start from the definition of conjugate momenta, which leads to primary constraints,

$$\Pi_{\bar{s}} \equiv \frac{\partial\mathcal{L}}{\partial(\partial_+\bar{s})} = -\frac{1}{2\kappa^2}\partial_- s\,, \tag{44}$$

$$\Pi_s \equiv \frac{\partial\mathcal{L}}{\partial(\partial_+ s)} = -\frac{1}{2\kappa^2}\partial_-\bar{s}\,. \tag{45}$$

There is no interaction dependent quadratic contribution to the canonical momenta, indicating that the quantization of the hamiltonian up to the cubic terms is straightforward. For an easier comparison with dilaton-axion $(\tau, \bar{\tau})$ system [4], we have divided the cubic hamiltonian (43) into two parts; the latter terms in the square bracket are Goroff-Schwarz terms [12, 4] that ensure the Lorentz invariance of the system when $f = 0$. We recall that in order to obtain the cubic terms for the dilaton-axion system, nonlocal field redefinitions

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(in ref. [4, eqs. (2.9), (2.10)] or (27)) were performed *ad hoc* to make sure the absence of the terms involving light-cone time derivatives in the cubic interaction. In the current analysis of the chiral primary system, similar expressions (25), (26) are a perturbative solution of the self-duality condition. The chiral primary system clearly has an advantage over the dilaton-axion system when it comes to the analysis of the higher order interactions.

We now come to demonstrate our main claim; the interaction hamiltonian in (43) is identical to that of the dilaton-axion system[3] given in [4], except for the relative minus sign between the terms involving ∂_i and the terms involving $\partial_{i'}$. The sign difference makes the hamiltonian (43) Z_2-invariant when we assign negative parity to s and $\bar{s}$. The quadratic term H_2 (42) indicates that the energy of the incoming ground state $|s\rangle_{p^+>0}$ (and the outgoing ground state $\langle\bar{s}|_{p^+<0}$) remains zero when we turn on f, while its complex conjugate incoming state $|\bar{s}\rangle_{p^+>0}$ (and the outgoing state $\langle s|_{p^+<0}$)and acquires the energy $8f$. It is also instructive to write down the interaction hamiltonian in the momentum basis. We perform the replacement

$$\frac{i}{\partial_-^r} \to \frac{1}{p_r^+} = \frac{1}{\alpha_r} \tag{46}$$

for the r-th particle to obtain

$$H_3 = \int d\mu_3 \, h_3(\alpha_r) s_1 s_2 \bar{s}_3 + \text{symmetrization} + (\text{c.c.})\,. \tag{47}$$

Here, the integration measure $d\bar{\mu}_3$ is given by

$$d\mu_3 = \left(\prod_{r=1}^{3} \frac{d\alpha_r}{2\pi}\frac{d^8 p_r}{(2\pi)^8}\right)\delta(\Sigma\alpha_r)\delta^8(\Sigma p_r)\,, \tag{48}$$

and

$$h_3 = \frac{1}{12\kappa^2}\frac{\alpha_1^4+\alpha_2^4+\alpha_3^4}{\alpha_1^2\alpha_2^2\alpha_3^2}\left(P_\perp^2 - P_\parallel^2\right), \tag{49}$$

where the 'momentum' P is expressed as

$$P = \alpha_1 p_2 - \alpha_2 p_1 = \alpha_2 p_3 - \alpha_3 p_2 = \alpha_3 p_1 - \alpha_1 p_3 \tag{50}$$

in terms of the transverse momentum p_r of the r-th particle. When $\alpha_1 > 0$, $\alpha_2 > 0$ and $\alpha_3 < 0$ such that $|\alpha_3| = |\alpha_1| + |\alpha_2|$, describing the interactions among two incoming chiral primary states and one outgoing chiral primary state, we have

$$H_3 \propto \int d\bar{\mu}_3 \left[\frac{(|\alpha_1|+|\alpha_2|+|\alpha_3|)^4}{\alpha_1^2\alpha_2^2\alpha_3^2}\left(P_\perp^2 - P_\parallel^2\right) s_1 s_2 \bar{s}_3\right]. \tag{51}$$

[3]A trivial rearrangement of $\tau = \phi + i\chi$ to remove the factor i in [4] is necessary. This way, the real component of τ becomes the NS-NS field just as the case of $s = h + ia$. In addition, the value of $\sqrt{\det g}$ for the background metric is two in ref. [4] and one in our case. We thus multiply (42) and (43) by the factor of two. After the rescaling $s \to \frac{\kappa}{\sqrt{2}}s$ and $\bar{s} \to \frac{\kappa}{\sqrt{2}}\bar{s}$, we get the number one in front of (42) and the factor $\sqrt{2}\kappa$ in front of (43), consistent with eqs. (2.15) and (2.16) of ref. [4].

The directions $\|$ and $\perp$ denote the four parallel directions (coming from AdS_5) and the four perpendicular directions (from S^5), respectively. Written in this way, we can compare our results with the (fermionic) prefactor part computed for the chiral primary fields in refs. [4, 5]. They agree with the light-cone string field theory analysis of [8] and are consistent with string bit theory[4] analysis of [14], where the choice of $v_{IJ} = \Delta_{IJ}$ (see (21)) was made for the comparison with the light-cone string field theory. An important remaining issue is to understand (43) from the dual gauge theory side [11]. The string bit theory analysis [14] indicates that a subtle basis change is necessary to find the map between gauge theory states and string theory states at the interaction level.

Finally one might wonder why the interaction hamiltonian for the chiral primary system is quite similar to that for the axion-dilaton system as obtained in [4]. For the axion-dilaton system, we just have to replace the factor $P_\perp^2 - P_\|^2$ in (51) by $P_\perp^2 + P_\|^2$. For the heuristic undestanding of this difference, one might recall that in the light cone analysis of the type-IIB string theory on the plane wave background, there are several choices of the fermionic vacua, which are related by the change of the labeling of the states [3]. One choice is SO(8) invariant vacuum, which corresponds to the axion-dilaton state. Chiral primary system corresponds to the vacuum which has $\mathrm{SO}(4)\times\mathrm{SO}(4)$ invariance. Now this difference is reflected in the factors mentioned above. For the complete understanding, one has to study how the string field theory results obtained in [8], which chooses the axion-dilaton vacuum, change as one chooses different fermionic vaccuum. The result obtained in the paper suggests that there should be simple relations between the string field theory results defined on the different fermionic vacua.

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